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Diseño y validación de un instrumento para la observación de las prácticas preprofesionales del futuro profesor de matemática

Design and validation of an observational instrument for the pre-professional practices of the future mathematics teacher

Salvador Alejandro Alarcón Godoy[a]
Carolina Henríquez-Rivas[b]

[a] Department of Basic Sciences, University of Concepción (UdeC), Chile, salvadoralarcon@udec.cl

[b] Faculty of Basic Sciences, Catholic University of Maule (UCM), Chile, chenriquezr@ucm.cl

Abstract

This study presents the design and validation of an instrument for observation during the preprofessional practicum of future mathematics teachers, through a process that involved five stages: (1) a literature review focused on two areas: the regulations that guide initial teacher education in Chile and theoretical frameworks that have been used in the field of mathematics education to analyze the work of teachers or future mathematics teachers; (2) preliminary design of the instrument, based on the categories of the conceptual framework, which included three dimensions, ten subdimensions, and twenty-six descriptors; (3) content validation by expert judgment, with internal consistency analyzed using Aiken's V coefficient; (4) adjustments to the preliminary design; and (5) final version of the instrument. For content validation, the instrument was evaluated by a panel of eight experts. Aiken's V coefficient was used for statistical analysis, showing acceptable levels (≥ 0.7) for most items. It is concluded that the use of this instrument will contribute to observing and providing feedback on the practices of future mathematics teachers and improving their initial training.

Keywords: professional development, instrument, pre-service practice, mathematics, teacher training

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Introduction

In recent decades, mathematics teacher education has become an object of study within the educational context (García, 2005; León et al., 2019). In particular, the study of the practical training of future teachers has emerged as a topic of interest within the discipline (Verdugo-Hernández et al., 2024). The study by Sánchez (2013) shows that initial teacher education (ITE) has become a subject of educational policy, while Michael Barber and Mona Mourshed (2007), as well as Pino-Fan et al. (2018), emphasize its importance for achieving quality educational processes in the classroom. For this reason, various organizations responsible for establishing educational policies have prioritized ITE in their recommendations (Beech, 2004; Beech, 2007). In particular, within mathematics teacher education, these recommendations focus on disciplinary and didactic aspects (UNESCO Santiago/OREALC, 2013).

Thus, the study of the practical training of future mathematics teachers constitutes an area of interest for educational research. Llinares (2018) highlights the need to conduct research in teacher education aimed at understanding practice, as well as the learning and professional development processes of mathematics teachers that support decision-making in both initial and continuing education programs.

Contextual Background: The Case of Initial Teacher Education in Chile

The Chilean legal framework establishes a National System for Quality Assurance in Higher Education (Law 20.129 of 2006). In the case of teacher education, this law establishes admission requirements, defines student assessment processes, and details guidelines for the National Accreditation Commission (CNA-Chile) to evaluate these programs. One of the criteria established by the CNA is linked to practical training, referring to the design, implementation, and monitoring of a formal system or organization for practical teacher preparation. This training component must enable future teachers to demonstrate the competencies, objectives, or learning outcomes outlined in the graduate profile and to show increasing development of teaching effectiveness within the training context (CNA-Chile, 2023).

In Chile, Initial Teacher Education (ITE) in Mathematics incorporates early field experiences (pre-professional practice) within public universities as part of teacher preparation. Its importance is reflected in the regulations governing ITE (Law 20.129 of 2006; CNA-Chile, 2023), where practical training is considered a relevant component of teacher education. This training axis includes several pre-professional practice courses throughout the educational trajectory. By way of illustration, Table 1 shows its incorporation into Mathematics Teacher Education programs.

Table 1. *Pre-professional Practice in Mathematics Teacher Education Programs*

Zona geográfica	Universidad	Número de prácticas preprofesionales	Semestre dentro del plan de estudio
Norte	De Antofagasta	4	2, 4, 6 y 8
Centro	Metropolitana de Ciencias de la Educación	5	2, 3, 5, 7 y 8
Sur	De La Frontera	4	4, 6, 7 y 8

Fuente: elaboración propia (extraído de: https://acceso.mineduc.cl).

These classroom experiences are positively valued by Chilean researchers of national and international recognition due to their contribution to the field of mathematics teacher education (Pino-Fan et al., 2018). Specifically, the early placement of future teachers in educational settings has gained strategic importance, making it essential that practical training (both professional and pre-professional) be the subject of research (Contreras et al., 2010). Other authors highlight the relevance of observing teaching practices within the process of initial teacher education in general (Fuertes, 2011; Arteaga-Martínez et al., 2021), insofar as such observation allows for analysis and reflection upon practice and becomes a tool for professional development.

Instruments for the Observation of Mathematics Teachers

The observation of mathematics classes has increasingly become a central tool for improving learning processes in classrooms; therefore, "it becomes necessary to develop instruments that allow us to frame observation and feedback, and to define what we are going to observe" (Martínez et al., 2018, p. 40). In this sense, different instruments have been designed for the observation of mathematics teachers, including: RTOP (Evaluation Facilitation Group, 2000); IQA (Boston & Wolf, 2006); M-Scan (Berry et al., 2012); Pauta MateO (Centro de Investigación Avanzada en Educación [CIAE], 2017); Manual ProMate (CIAE et al., 2018); and POEMat.ES (Joglar et al., 2021). However, these instruments are not adapted to mathematics classroom observation within the context of practical teacher education (Boston et al., 2015). Likewise, Espinoza et al. (2023) highlight the growing interest in the construction of instruments for classroom observation.

The above demonstrates the relevance of the practicum component within teacher education, specifically in mathematics education, as well as the reported scarcity of instruments designed to observe classroom performance, particularly among future teachers conducting formative activities in real school settings. Therefore, this article proposes as its objective the design and validation of an instrument for observing the pre-professional practices of future mathematics teachers.

The following sections present the theoretical, methodological, and empirical foundations supporting the development of the instrument. First, the conceptual framework is presented, articulating normative and theoretical references from pedagogical, disciplinary, and didactic-disciplinary domains. Subsequently, the methodology employed for its design and validation is described, based on expert judgment and analysis using Aiken's V coefficient. The results are organized around the different stages of the instrument's construction and evaluation process, highlighting the adjustments made based on the qualitative feedback received. Finally, the implications of the instrument for initial teacher education are discussed, and directions for future research are proposed.

Theoretical Foundation

This framework is based on specialized literature on teacher education and teachers' specific knowledge, approached from two perspectives: (1) literature that guides initial teacher education, and (2) theoretical reference frameworks that have been used in the field of

mathematics education to analyze the work of mathematics teachers or future mathematics teachers. These perspectives have been organized according to three domains: pedagogical, disciplinary, and didactic-disciplinary (Lee Shulman, 1986; Arteaga-Martínez et al., 2021; Van Zoest et al., 2021).

Regarding the review of current documentation and regulations in Chile, it can be noted that the Teaching Profession Standards (EPD) constitute the principal framework defining the pedagogy that children and young people need in order to develop their full potential (Centro de Perfeccionamiento, Experimentación e Investigaciones Pedagógicas [CPEIP], 2021a). These standards are projected in relation to two stages of the teaching profession: (1) initial teacher education, through the Pedagogical and Disciplinary Standards for Teacher Education Programs (hereafter, Initial Teacher Education Standards) (CPEIP, 2021b); and (2) professional practice, through the Teaching Profession Performance Standards contained in the Framework for Good Teaching (CPEIP, 2021c). In particular, the Initial Teacher Education Standards (Ministry of Education of Chile, 2017) are conceived as guidelines that explicitly define the set of skills, knowledge, and dispositions that an education professional should possess upon completing initial teacher education within both pedagogical and disciplinary domains (CPEIP, 2021a).

With regard to disciplinary and didactic-disciplinary aspects, two theoretical models are considered: (1) the Mathematics Teacher's Specialized Knowledge model (MTSK) (Carrillo-Yañes et al., 2018), which has been widely used to analyze mathematics teachers' knowledge (Martignone et al., 2022; Loría-Fernández et al., 2023); and (2) the Theory of Mathematical Working Spaces (MWS) (Kuzniak et al., 2022), applied to analyze the mathematical work of both teachers and students (Henríquez-Rivas & Kuzniak, 2021). Both perspectives have been used in a complementary manner, as they allow for deeper examination of the didactic and disciplinary aspects of mathematics teachers' practice (Espinoza-Vásquez et al., 2025; Verdugo-Hernández & Caviedes-Barrera, 2024). The dimensions considered for the design of the instrument are described below:

Pedagogical Dimension

According to CPEIP (2021b), the pedagogical domain is understood as the teaching competencies required across all subject areas to ensure high-quality pedagogical interactions that support student learning. These competencies encompass the knowledge, skills, and dispositions that teachers are expected to master so that their students can actively participate in educational processes.

According to CPEIP (2021b), these competencies are evidenced through practices that engage students in cognitively challenging tasks; formative and summative assessment used to adjust instruction and provide feedback; the promotion of well-being and a positive school climate; explanations that clarify content more effectively; modeling and the use of examples; the facilitation of classroom discussion; the promotion of thinking skills and problem-solving; and the use of information and communication technologies (ICT). These practices may be organized into five aspects: (1) a favorable and safe environment, (2) organized strategies for

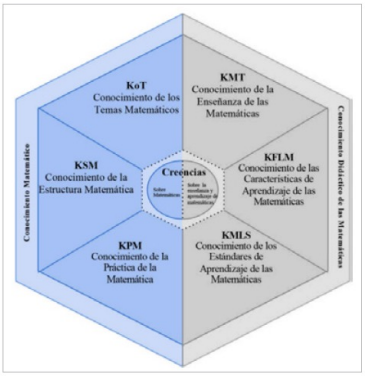
teaching, assessment, and feedback, (3) promotion of skill development through social and emotional learning, (4) development of teaching capacities, and (5) support for students.

Disciplinary Dimension

The disciplinary dimension is understood, according to its conceptualization within the Initial Teacher Education Standards (FID Standards), as what graduates must demonstrate regarding mastery of the knowledge specific to their discipline (CPEIP, 2021b). In this regard, it is stated that excellence in teaching rests upon a deep interconnection between content knowledge and the pedagogical capacity to generate representations, actions, and reflections concerning such knowledge (Lee Shulman, 1986).

The MTSK model (Mathematics Teacher Specialized Knowledge) makes it possible to focus on teachers' understanding (Carrillo-Yañes et al., 2018), following the work of Shulman (1986) and Ball et al. (2008). It has been applied across different educational levels (primary, secondary, and higher education), both with practicing and pre-service teachers (Advíncula-Clemente et al., 2020), revealing its usefulness as a theoretical and analytical tool that enables the identification of mathematics teachers' specialized knowledge and supports understanding its nature in a systematic and organized way. The model may be understood through two domains of knowledge: mathematical knowledge and didactic-mathematical knowledge. In the present study, the focus is on examining the mathematical knowledge (MK) transmitted by the teacher and the way in which it is enacted (Advíncula-Clemente et al., 2020). Figure 1 presents the MTSK model, where the blue section corresponds to the approach considered in this study.

Figure 1. *Diagram of the MTSK Model*



Fuente: Carrillo-Yañez et al. (2018, p. 241).

The MK domain considers three subdomains: Knowledge of Topics (KoT), which refers to what mathematics teachers know and how they know the topics they teach, including the procedures involved, representation registers, related uses and applications, definitions, properties, and their underlying principles; Knowledge of the Structure of Mathematics (KSM), which corresponds to teachers' knowledge of the connections among mathematical topics, either within the same topic or across different topics; and Knowledge of Mathematical Practice (KPM), which includes knowledge about how mathematics is created, carried out, and produced, considering rules for constructing knowledge, as well as proof and problem-solving processes (Advíncula-Clemente et al., 2020; Zakaryan & Soza, 2021). In this way, the proposed instrument is related to these three MTSK subdomains.

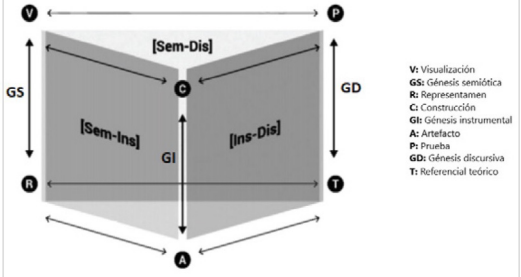
Didactic-Disciplinary Dimension

The didactic-disciplinary domain is related to the specific knowledge associated, in this case, with mathematics and its teaching, involving both mastery of disciplinary knowledge and didactic knowledge for its instruction. The Theory of Mathematical Working Spaces has been widely used to analyze teachers' mathematical work in the classroom (Henríquez-Rivas et al., 2022; Henríquez-Rivas & Verdugo-Hernández, 2023), as it contributes to understanding teachers' practices when carrying out mathematical tasks and makes it possible to characterize the pathways that emerge during their resolution (Kuzniak et al., 2022).

At the epistemological level are the principles inherent to mathematical objects in operation, while at the cognitive level lies the thinking of the individual who uses these mathematical objects to solve a task. The articulation between these two levels is established through three genesis processes: semiotic genesis, which articulates the representamen, considering the mathematical objects involved and the visualization processes linked to the semiotic representation of mathematical objects; instrumental genesis, which connects artifacts that may be material, technological, or symbolic tools used in the application of objects; and discursive genesis, which articulates the theoretical framework and proofs, encompassing the entire discursive process of validation, including definitions, hypotheses, conjectures, counterexamples, and deductive arguments (Kuzniak et al., 2016).

In turn, the different interactions that may occur between two genesis processes and their associated components are represented through vertical planes (Kuzniak & Richard, 2014). These vertical planes connect different phases of mathematical work and the meanings involved in the execution of a task (Gómez-Chacón et al., 2016), as represented in Figure 2. The semiotic-discursive vertical plane [Sem-Dis] involves coordinating the visualization process of represented objects with reasoning oriented toward mathematical communication. On the other hand, the semiotic-instrumental vertical plane [Sem-Ins] involves the use of artifacts for the construction of results or the exploration of objects through semiotic representations that enable the development of competencies associated with discovery. Finally, the instrumental-discursive vertical plane [Ins-Dis] relates proofs based on experimentation, the validation of a construction, or empirical generalization with the use of an artifact (Kuzniak & Richard, 2014; Kuzniak et al., 2016).

Figure 2. *Diagram of the Planes, Genesis Processes, and Components of Mathematical Working Spaces (MWS)*



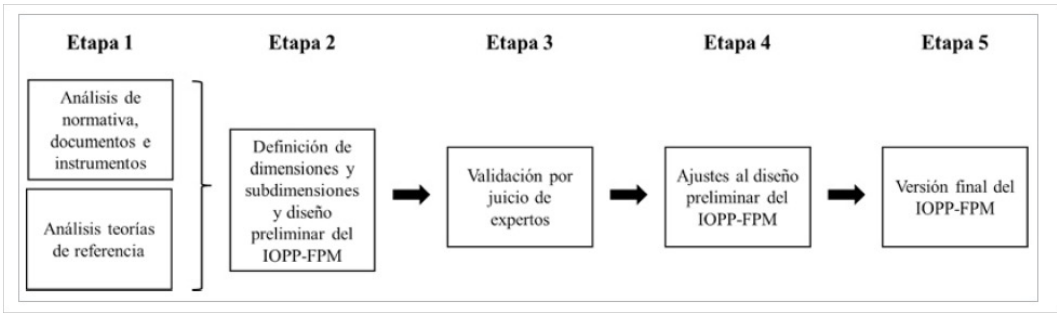
Fuente: adaptado de Kuzniak et al. (2016, p. 864).

This research places particular emphasis on the current optimal Mathematical Working Space (MWS) of the future teacher, understood as the study of the mathematical work that is observed and that actually takes place in the classroom during the process of teaching mathematics (Henríquez-Rivas et al., 2022).

Methodology

The present study presents the design and content validation of an instrument called the Instrument for Observing the Pre-Professional Practice of Future Mathematics Teachers (IOPPFPM). It is a mixed-methods study (John W. Creswell & Creswell, 2023), since the first stage involves qualitative studies for the construction of the instrument, followed by a quantitative stage within the validation process. In addition, a descriptive study approach is adopted (Lederman, 1993), as the study focuses on constructing and evaluating the characteristics and psychometric properties of the instrument through content analysis based on expert judgment (Escobar-Pérez & Cuervo-Martínez, 2008). To achieve this, Aiken's V coefficient is used (Aiken, 1985), allowing the analysis of the internal consistency of the descriptors' content. This process comprises five stages, which are presented in Figure 3.

Figure 3. *Diagram of the Construction and Validation Process of the IOPP-FPM*



In Stage 1, a literature review was conducted, focusing on the regulations guiding initial teacher education in Chile and the theoretical reference frameworks that had been used in the field of mathematics education to analyze teachers' work. These topics were addressed considering several reference frameworks, among them: (i) from the pedagogical perspective, the Pedagogical and Disciplinary Standards for Mathematics Teacher Education Programs (CPEIP, 2021b) and the Framework for Good Teaching (CPEIP, 2021c); (ii) from the disciplinary perspective, the MTSK model (Carrillo-Yañes et al., 2018); and (iii) from the didactic-disciplinary perspective, the Theory of Mathematical Working Spaces (MWS) (Kuzniak et al., 2022).

In Stage 2, the instrument was preliminarily designed according to the categories established within the conceptual framework. In Stage 3, the validation process was carried out with the participation of seven experts from three Chilean universities. All participants were specialists linked to the initial and continuing education of mathematics teachers. Five of them held PhDs in Mathematics Education, one held a Master's degree in Mathematics Education, and another held a PhD in Mathematics.

The experts received an invitation by email explaining the context, description, and purpose of the study, together with a guide for evaluating the initial 26 descriptors according to their clarity (whether they are understandable), pertinence (whether they are related to the subdimension being measured), and relevance (whether they should be included according to the research objective). A Likert-type scale was used to evaluate the descriptors, with scores ranging from 1 to 4, where 1 indicated noncompliance with the declared definition of the evaluated criterion, and 4 indicated that the descriptor fully complied with the declared definition of the evaluated criterion. In addition, the experts were asked to evaluate the sufficiency of the descriptors in relation to the corresponding subdimension and to include qualitative observations that would allow improvement of those receiving low scores. Likewise, a section was included for assessing qualitative aspects from their perspective as experts.

In Stage 4, adjustments were made both to the instrument as a whole and to the descriptors whose Aiken's V coefficient was not at an acceptable level, that is, below 0.7 (Charter, 2003), taking into consideration the qualitative suggestions received. Finally, Stage 5 involved the final design of the instrument.

Results

The results of the design and validation process of the IOPP-FPM are presented below, organized according to the five stages previously described.

Stage 1: Literature Review

With regard to the pedagogical domain, general elements were identified that are not exclusive to school mathematics as a subject, but rather transcend different curricular disciplines. Regarding the disciplinary and didactic-disciplinary domains, mathematical disciplinary aspects and activities that demonstrate mathematical work within the classroom context were identified.

Stage 2: Preliminary Version of the Instrument

The conceptual framework made it possible to define three dimensions and ten associated subdimensions of the instrument. The Pedagogical Dimension includes three subdimensions: a learning-conducive environment, monitoring of students' work, and the deployment of socio-emotional variables. The Disciplinary Dimension includes three subdimensions: knowledge of mathematical topics, knowledge of mathematical structure, and knowledge of mathematical practice. The Didactic-Disciplinary Dimension encompasses four subdimensions: semiotic genesis, instrumental genesis, discursive genesis, and vertical planes.

Based on these subdimensions, 26 descriptors were developed, each oriented toward a specific subdimension. The wording of the descriptors was designed for the observation of the future mathematics teacher's classroom work by the supervisor, who is the university professor responsible for supervising, supporting, and evaluating future teachers within the context of their practicum in an educational institution (Hirmas, 2014). Each descriptor

is accompanied by a Likert-type scale: 0 (not applicable), 1 (rarely observed), 2 (generally observed), 3 (frequently observed), and 4 (consistently observed).

Once the 26 descriptors had been defined, the process of content validation was conducted for each of them, according to the dimensions of the instrument.

Table 2 below presents the organization of the instrument in its preliminary version, according to dimension, subdimension, and number of descriptors.

Table 2. *Summary of the Dimensions and Subdimensions of the Preliminary Instrument*

Dimensiones	Subdimensiones	Cantidad de descriptores	Características generales
Dimensión pedagógica	S1: Ambiente propicio para el aprendizaje	3	Relacionadas con las competencias docentes que aseguren interacciones pedagógicas de calidad para el logro de aprendizajes.
	S2: Monitoreo del trabajo de los estudiantes	3	
	S3: Despliegue de variables socioemocionales	3	
Dimensión disciplinar	S4: Conocimiento de los temas matemáticos	3	Relacionados con el conocimiento matemático que el futuro profesor usa o puede usar en cualquier actividad respecto a los temas que enseña y la forma en que los conoce.
	S5: Conocimiento de la estructura matemática	3	
	S6: Conocimiento de la práctica matemática	3	
Dimensión didáctica disciplinar	S7: Génesis Semiótica	2	Relacionados con las componentes, génesis y planos verticales descritos en los ETM, a fin de comprender aspectos del trabajo matemático en el aula realizado por el futuro profesor de matemática desde una perspectiva didáctica.
	S8: Génesis Instrumental	2	
	S9: Génesis Discursiva	3	
	S10: Planos verticales	3	

Stage 3: Validation Through Expert Judgment

The results of the average scores assigned to the descriptors and Aiken's V coefficient for each of the evaluated criteria are presented below.

Clarity

The clarity of 25 out of the 26 descriptors was evaluated very favorably by the expert validators. Table 3 presents the scores corresponding to the third descriptor within each of the dimensions:

Table 3. *Mean Scores and Aiken's V Coefficients in the Pedagogical Dimension*

Subdimensión	S1			S2			S3	
Descriptor	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Media	3,143	3,286	2,143	3,286	3,286	3,714	4	4
V de Aiken	0,714	0,762	0,381	0,762	0,762	0,905	1	1

Descriptor 3 requires special attention regarding clarity. Its mean score is the lowest within the dimension, and even within the entire instrument (as can be observed in other tables). In addition, its coefficient falls below the cutoff value mentioned previously. The remaining descriptors show mean scores above 3 and coefficients at acceptable levels. In particular, descriptors 7 and 8 obtained maximum mean scores and ideal coefficients.

Table 4. *Mean Scores and Aiken's V Coefficients in the Disciplinary Dimension*

Subdimensión	S4			S5			S6		
Descriptor	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
Media	3,714	3,571	3,571	3,714	3,571	3,571	3,714	3,857	3,714
V de Aiken	0,905	0,857	0,857	0,905	0,857	0,857	0,905	0,952	0,905

As shown in Table 4, all descriptors associated with this dimension present mean scores above 3.5 and coefficients higher than 0.8; therefore, according to the judges, their clarity is adequate and does not require further revision.

Table 5. *Mean Scores and Aiken's V Coefficients in the Didactic-Disciplinary Dimension*

Subdimensión	S7		S8		S9		S10		
Descriptor	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Media	3,429	3,571	3,714	3,429	3,857	4	3,286	3,286	3,571
V de Aiken	0,81	0,857	0,905	0,81	0,952	1	0,762	0,762	0,857

In this dimension, as in the previous one, all descriptors show mean scores above 3 and coefficients higher than 0.8; in particular, descriptor 23 achieved the maximum score and coefficient (Table 5). Thus, the clarity of these descriptors is considered adequate and does not require further revision. Overall, according to the clarity criterion, the review process will focus on the wording of descriptor 3, considering the mean score and coefficient obtained, in order to make the corresponding adjustments based on the qualitative suggestions provided by the judges.

Pertinence

The pertinence of 25 out of the 26 descriptors was evaluated very favorably by the consulted experts.

Table 6. *Mean Scores and Aiken's V Coefficients in the Pedagogical Dimension*

Subdimensión	S1			S2			S3	
Descriptor	1	2	3	4	5	6	7	8
Media	4	4	3,5	3,857	4	3,714	4	4
V de Aiken	1	1	0,833	0,952	1	0,905	1	1

As shown in Table 6, the descriptors associated with this dimension present mean scores greater than or equal to 3.5 and coefficients at acceptable levels. Descriptors 1, 2, 5, 7, and 8 obtained maximum mean scores and ideal coefficients. Considering these data, the descriptors do not require further revision according to this criterion.

Table 7. *Mean Scores and Aiken's V Coefficients in the Disciplinary Dimension*

Subdimensión	S4			S5			S6		
Descriptor	9	10	11	12	13	14	15	16	17
Media	3,714	3,877	3,714	3,714	3,714	3,714	3,571	3,571	3
V de Aiken	0,905	0,952	0,905	0,905	0,905	0,905	0,857	0,857	0,667

According to Table 7, descriptor 17 obtained the lowest mean score within the dimension, together with a coefficient below the critical acceptance value defined in this study. The remaining descriptors associated with this dimension show mean scores above 3.5 and coefficients higher than 0.8; therefore, according to the judges, the pertinence of these descriptors is considered adequate and they do not require further revision.

Table 8. *Mean Scores and Aiken's V Coefficients in the Didactic-Disciplinary Dimension*

Subdimensión	S7		S8		S9		S10		
Descriptor	18	19	20	21	22	23	24	25	26
Media	3,857	3,857	4	3,714	4	4	4	3,667	3,429
V de Aiken	0,952	0,952	1	0,905	1	1	1	0,889	0,81

In this dimension, as in the previous one, all descriptors show high mean scores above 3 and coefficients higher than 0.8. Descriptors 20, 22, 23, and 24 achieved maximum mean scores and coefficients (Table 8). Thus, the pertinence of these descriptors is considered adequate

and, according to the judges, they do not require further revision. Regarding the pertinence criterion, the review process focuses on descriptor 17 in order to make the corresponding adjustments based on the qualitative observations provided by the judges.

Relevance

All 26 descriptors of the instrument were positively evaluated by the judges. In fact, this criterion obtained the highest scores and coefficients.

Table 9. *Mean Scores and Aiken's V Coefficients in the Pedagogical Dimension*

Subdimensión	S1			S2			S3	
Descriptor	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Media	4	4	3,667	3,857	4	3,714	4	4
V de Aiken	1	1	0,889	0,952	1	0,905	1	1

Table 9 shows that the descriptors associated with this dimension present mean scores greater than or equal to 3.6 and coefficients at acceptable levels, higher than 0.8. Meanwhile, descriptors 1, 2, 5, 7, and 8 obtained maximum mean scores and ideal coefficients. Considering these data, the descriptors do not require further revision according to this criterion.

Table 10. *Mean Scores and Aiken's V Coefficients in the Pedagogical Dimension*

Subdimensión	S4			S5			S6		
Descriptor	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
Media	4	4	3,571	3,571	3,571	3,429	3,714	3,857	3,286
V de Aiken	1	1	0,857	0,857	0,857	0,81	0,905	0,952	0,762

The descriptors associated with this dimension show mean scores above 3 and coefficients higher than 0.7, including descriptors 9 and 10, which obtained maximum scores and coefficients. Therefore, their relevance is considered adequate and they do not require further revision (Table 10).

Table 11. *Mean Scores and Aiken's V Coefficients in the Didactic-Disciplinary Dimension*

Subdimensión	S7		S8		S9		S10		
Descriptor	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Media	3,857	3,857	4	3,714	4	4	4	3,667	3,429
V de Aiken	0,952	0,952	1	0,905	1	1	1	0,889	0,81

All descriptors associated with this dimension show mean scores above 3 and coefficients higher than 0.8. As in the pertinence criterion, descriptors 20, 22, 23, and 24 achieved maximum mean scores and coefficients (Table 11). In this way, the relevance of these descriptors is considered adequate and does not require further revision. Regarding the relevance criterion, no revision of the descriptors will be necessary, since both the mean scores and the coefficients are above the level defined as acceptable.

Sufficiency

According to the judges' evaluations, the presented descriptors are considered sufficient for the respective dimensions to which they are associated. The mean scores and coefficients for the instrument are presented in Table 12.

Table 12. *Mean Scores and Aiken's V Coefficients for the Sufficiency Criterion*

Dimensión	D1	D2	D3
Media	3,333	3,5	3,667
V de Aiken	0,778	0,833	0,889

In summary, the review process focuses on descriptors 3 and 17, according to the experts' evaluations and considering their qualitative observations.

Stage 4: Adjustments to the Instrument

According to the results of the content validation, the instrument received a highly positive evaluation from the experts. The criteria of clarity, pertinence, relevance, and sufficiency obtained mean scores close to the maximum and Aiken's V coefficients at acceptable levels. Nevertheless, descriptors 3 and 17 required revision for clarity and pertinence, respectively. In addition, comments from three experts suggested incorporating a section for qualitative observations so that the supervising teacher would have the opportunity to record other elements that might not be contemplated within the descriptors.

Adjustments to Descriptor 3

The observations provided by the experts indicated the need to clarify the wording of the descriptor:

- "Revise the wording" (Evaluator Judge 4)
- "It is not clear what it is trying to say" (Evaluator Judge 7)

Likewise, the qualitative suggestions proposed a new wording for the descriptor:

- "It could follow the tone of the former Framework for Good Teaching: responds assertively to breaches of classroom coexistence rules" (Evaluator Judge 2)
- "Manages students' behavior according to the established classroom coexistence rules" (Evaluator Judge 3)

The authors proposed a new wording:

"Manages students' behavior assertively and effectively in response to breaches of classroom coexistence rules."

Adjustments to Descriptor 17

The observations provided by the experts suggested eliminating the descriptor. According to their comments:

- "It is similar to descriptor 11. They would be evidencing the same thing" (Evaluator Judge 1)
- "This was already addressed in a previous descriptor" (Evaluator Judge 2)

For this reason, descriptor 17 was removed, since it was already included in another descriptor (descriptor 11).

Other Adjustments

The experts agreed on suggesting the incorporation of a qualitative observations section within each subdimension, with the purpose of allowing the supervising teacher to report any other relevant aspect that might not be included in the descriptors.

- "It would be good to include a space associated with the dimensions to write down other emerging aspects" (Evaluator Judge 2)
- "A space for observations is missing" (Evaluator Judge 4)

Stage 5: Final Version of the Instrument

The final version of the IOPP-FPM instrument, after the adjustments related to the validation process and the qualitative suggestions provided by the experts, includes 25 descriptors organized into dimensions and subdimensions. Each descriptor is associated with an assessment scale in which the supervising teacher must identify whether each aspect is observed rarely, generally, frequently, or consistently during the development of the class, or whether it is not applicable. In addition, a section for qualitative observations is included at the end of each subdimension. The descriptors organized by subdimension are presented in Table 13.

Table 13. *Validated Descriptors of the IOPP-FPM According to Subdimension and Dimension*

Dimensiones	Subdimensiones	Descriptores
Dimensión pedagógica	Ambiente propicio para el aprendizaje	1. Establece normas de funcionamiento de la clase, las que son comunicadas y reforzadas durante su desarrollo. 2. Promueve la participación equitativa de sus estudiantes en las actividades de la clase. 3. Gestiona asertiva y efectivamente el comportamiento de los estudiantes frente a las faltas a las normas de convivencia.

Dimensiones	Subdimensiones	Descriptores
Dimensión pedagógica	Monitoreo del trabajo de los estudiantes	4. Realiza preguntas retóricas o dirigidas que permiten verificar la comprensión de instrucciones y actividades, por parte de sus estudiantes. 5. Realiza preguntas retóricas o dirigidas que permiten verificar la comprensión de aprendizajes, por parte de sus estudiantes. 6. Propone tareas breves que permiten verificar la comprensión de aprendizajes por parte de sus estudiantes.
	Despliegue de variables socioemocionales	7. Promueve el desarrollo de habilidades sociales, como la escucha, el respeto por los turnos, la empatía u otros, en las interacciones entre pares. 8. Promueve el desarrollo de habilidades sociales, como la escucha, el respeto por los turnos, la empatía u otros, en las interacciones con el profesor.
Dimensión disciplinar	Conocimiento de los temas matemáticos	9. Muestra dominio de los conceptos y proposiciones matemáticas que enseña. 10. Muestra dominio de las propiedades de los objetos matemáticos que utiliza, aplicándolas de manera correcta. 11. Usa distintos ejemplos para la enseñanza del tema que presenta.
	Conocimiento de la estructura matemática	12. Relaciona el aprendizaje de la clase con otros conocimientos matemáticos, dentro del mismo tópico abordado. 13. Relaciona el aprendizaje de la clase con otros conocimientos matemáticos, dentro de otros tópicos distintos al abordado, en el mismo eje temático. 14. Relaciona el aprendizaje de la clase con otros conocimientos matemáticos, dentro de otros tópicos distintos al abordado, en otro(s) eje(s) temático(s).
	Conocimiento de la práctica matemática	15. Usa las definiciones, proposiciones y propiedades, que involucran los temas enseñados. 16. Demuestra o justifica las producciones matemáticas que enseña, considerando la lógica argumental detrás de su uso.
Dimensión didáctica disciplinar	Génesis semiótica	17. Usa distintas representaciones semióticas. 18. Las representaciones usadas permiten visualizar el objeto matemático en cuestión.

Dimensión didáctica disciplinar	Génesis instrumental	19. Usa distintos recursos materiales o tecnológicos, de acuerdo con los procedimientos correspondientes, para resolver o ejemplificar una tarea. 20. Usa teoremas o propiedades como algoritmos o fórmulas para resolver o ejemplificar una tarea.
	Génesis discursiva	21. Promueve el uso de conjeturas frente a distintas tareas de aprendizaje en la clase. 22. Promueve el uso de argumentaciones deductivas o inductivas frente a distintas tareas de aprendizaje en la clase.
	Planos verticales	23. Coordina la visualización de los objetos matemáticos que enseña, con un razonamiento que permite su comunicación matemática. 24. Se apoya en la visualización para identificar o explorar objetos que permiten resolver un problema o realizar una prueba. 25. Justifica las construcciones matemáticas realizadas.

Discussion and Conclusions

This article presents the phases that led to the design and validation of the instrument called IOPP-FPM for observing the practices of future mathematics teachers during their early field placement, while also emphasizing the importance of observing student teachers during their pre-professional practice for purposes of evaluation and feedback (Fuertes, 2011; Verdugo-Hernández & Caviedes-Barrera, 2024), as well as the need for an instrument capable of guiding such observation (Contreras et al., 2010).

The literature review conducted, based on regulatory documents within the context of Initial Teacher Education (ITE) in Chile, together with theoretical frameworks frequently used to analyze the knowledge and practice of future mathematics teachers, made it possible to organize the instrument into three dimensions, each with three or four subdimensions. The theoretical definitions of these dimensions supported the drafting of the descriptors. These descriptors were subsequently validated in terms of content through the judgment of seven experts in Mathematics or Mathematics Education, and the obtained scores were statistically analyzed using Aiken's V coefficient. The results of the expert evaluations show an overall positive assessment regarding the clarity, pertinence, relevance, and sufficiency of the descriptors, together with several qualitative suggestions provided by the judges. The statistical analysis, along with the adjustments made based on these suggestions, enabled the proposal of the final version of the IOPP-FPM, which is presented as a contribution toward achieving a more comprehensive instrument.

The development of this instrument makes it possible to obtain evidence regarding the practices carried out by future mathematics teachers during their early training through pre-professional practice activities. Through the information obtained from the IOPP-FPM,

it becomes possible to provide feedback to future teachers and, at the same time, make decisions at the level of teacher education, with particular attention to pre-professional practice activities.

Unlike other observation instruments (Evaluation Facilitation Group, 2000; Boston & Wolf, 2006; Berry et al., 2012; CIAE, 2017; CIAE et al., 2018; Joglar et al., 2021), whose design is oriented toward observing in-service teachers, the IOPP-FPM integrates the pedagogical dimension based on current Chilean regulations related to Initial Teacher Education (CPEIP, 2021b; 2021c), together with the disciplinary and didactic-disciplinary dimensions derived from theoretical frameworks (Carrillo-Yañez et al., 2018; Kuzniak et al., 2022) that are currently used complementarily within mathematics education research in order to deepen understanding of the (future) teacher during initial teacher education.

Regarding the theoretical aspects supporting this study, future research could consider the didactic-disciplinary subdomain of the MTSK model (Carrillo-Yañez et al., 2018) and specificities of the Mathematical Working Spaces theory (Kuzniak et al., 2022), which could strengthen the instrument and broaden its application within other teacher education contexts. Likewise, findings emerging from studies connecting these two theories could also be considered (Espinoza-Vásquez et al., 2025).

With respect to the limitations of this study, the number of expert judges participating in the content validation may have influenced the obtained results, since a larger number of evaluators provides greater flexibility in the coefficient value used (Escurra, 1988). Nevertheless, the academic backgrounds of the experts and their connection to the initial preparation of mathematics teachers support confidence in the final instrument obtained.

Finally, the designed and validated IOPP-FPM instrument is made available for use in observing the pre-professional practices of future mathematics teachers during their training. It is expected that the use of this instrument will allow its application within initial teacher education, support future construct validation processes, and contribute to defining profiles or categories of future mathematics teachers within the context of their early practical training, thereby helping to improve teacher education.

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